

TRANSITION TO COLLEGE MATHEMATICS AND STATISTICS A PROBLEM-BASED, TECHNOLOGY-RICH CAPSTONE COURSE FOR NON-STEM STUDENTS

Christian Hirsch

Western Michigan University

christian.hirsch@wmich.edu

This paper describes a four-year project in the U.S., funded by the National Science Foundation, to design, develop, and evaluate Transition to College Mathematics and Statistics (TCMS), an innovative senior-level course to help meet the diverse quantitative needs of students whose intended undergraduate programs do not require calculus (e.g., business; management; economics; the information, life, health, and social sciences; and many teacher preparation programs). For students intending to enroll in non-STEM undergraduate programs, many schools in the U.S. have little to offer as a transition to college-level mathematics and statistics other than Precalculus or narrow Advanced Placement courses. Consequently, many students opt out of mathematics their senior year or study mathematics that is inappropriate for their undergraduate and career aspirations. TCMS focuses on contemporary topics, including mathematical modeling, data analysis and inference, informatics, financial mathematics, decision-making under constraints, mathematical visualization and representations, and important mathematical habits of mind. TCMS is accompanied by TCMS-Tools, a concurrently developed suite of curriculum-embedded Java-based software, including a spreadsheet, a CAS, dynamic geometry, data analysis, simulation, and discrete mathematics tools together with specialized apps. The focus of the paper is on the TCMS content and its organization, pedagogical design, affordances of TCMS-Tools, and preliminary evaluation results from classroom trials of TCMS use as a capstone course in diverse settings.

Key Words: curriculum, mathematics, statistics, modeling, curriculum-embedded software

INTRODUCTION

Transition to College Mathematics and Statistics (TCMS) is a problem-based, inquiry-oriented, and technology-rich fourth-year high school mathematics course. It was developed to help ensure student success in college and careers in an increasingly technological, information-laden, and data-driven global society. *TCMS* was specifically designed for the large numbers of non-STEM oriented students whose undergraduate programs of study do not require calculus—such as business; management; economics; the information, life, health, and social sciences; and many teacher preparation programs. See Figure 1 for the top 10 undergraduate majors in the U.S., as reported in the *Princeton Review* (2014). Students intending to complete one of these majors or other non-STEM majors in college are often ill-prepared by the lack of appropriate fourth-year high school mathematics courses. Research has repeatedly shown that students who are not enrolled in an appropriate mathematics course their senior year are much more likely to be placed in a remedial (non-credit bearing) course in college (cf. Key & O'Malley, 2014). It is in this context that *TCMS* was designed, developed, and evaluated.



Figure 1. The case for an alternative to Precalculus as a fourth-year course.

OVERVIEW

Transition to College Mathematics and Statistics is designed to be used as a fourth-year capstone course for students who have successfully completed a conventional single-subject sequence of algebra, geometry, and advanced algebra or a three-year international-like integrated mathematics sequence. The course has been carefully field tested in high schools with students using conventional mathematics curricula and with students using an integrated mathematics program.

TCMS builds upon the theme of mathematics as sense-making. Through investigations of real-life contexts and problems, students develop a rich understanding of important mathematics that makes sense to them and that, in turn, enables them to make sense out of new situations and problems. This theme of sense-making as well as the pervasive expectation that students reason about mathematics align well with the recent National Council of Teachers of Mathematics (NCTM, 2014) recommendations for high school mathematics.

Key themes and instructional features as outlined below have been informed by research on student learning (cf. National Research Council, 2005; NCTM, 2014) and recommendations from client disciplines on the focus of undergraduate non-calculus-based mathematics and statistics courses (cf. Ganter & Barker, 2004).

Balanced Content—*Transition to College Mathematics and Statistics* reviews and extends students’ understanding of important and broadly useful concepts and methods from algebra and functions, statistics and probability, discrete mathematics, and geometry. These branches of mathematics are connected by the central theme of modeling our world and by mathematical habits of mind such as visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, exploiting use of multiple representations, providing convincing explanations, and a disposition towards strategic use of technological tools.

Flexibility—*TCMS* consists of eight focused and coherent units, each of which is generally self-contained with attention to content prerequisites provided by “Just-in-Time” review

tasks in lesson homework sets. The course has been organized to be as flexible as possible. The organization permits teachers to tailor courses that best meet the needs and interests of their students. For example, some teachers choose to use the unit on *Mathematics of Democratic Decision-Making* as the second or third unit of the course to parallel state or national elections in the U.S.

Mathematical Modeling—*TCMS* emphasizes mathematical modeling including the processes of problem formulation, data collection, representation, interpretation, prediction, and simulation. Problem situations and phenomena modelled in this course involve discrete and continuous variables and entail deterministic as well as stochastic processes.

Technology—Numeric, graphic, and symbolic manipulation capabilities such as those found in *TCMS-Tools*® and on many graphing calculators are assumed and appropriately used throughout the course. *TCMS-Tools* is a suite of software tools that provide powerful aids to learning mathematics and solving mathematical problems. This use of technology permits the curriculum and instruction to emphasize multiple linked representations (verbal, numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking and problem solving are central. Figure 2 provides three sample screens from the geometric linear programming (LP) custom app in which, in this case, students formulate and then enter the constraint inequalities and the objective function for two-variable linear programming problems. The LP app also supports solutions of three-variable linear programming problems. The software enables the student to dynamically control the line (plane) representing the objective function in search of an optimal solution.

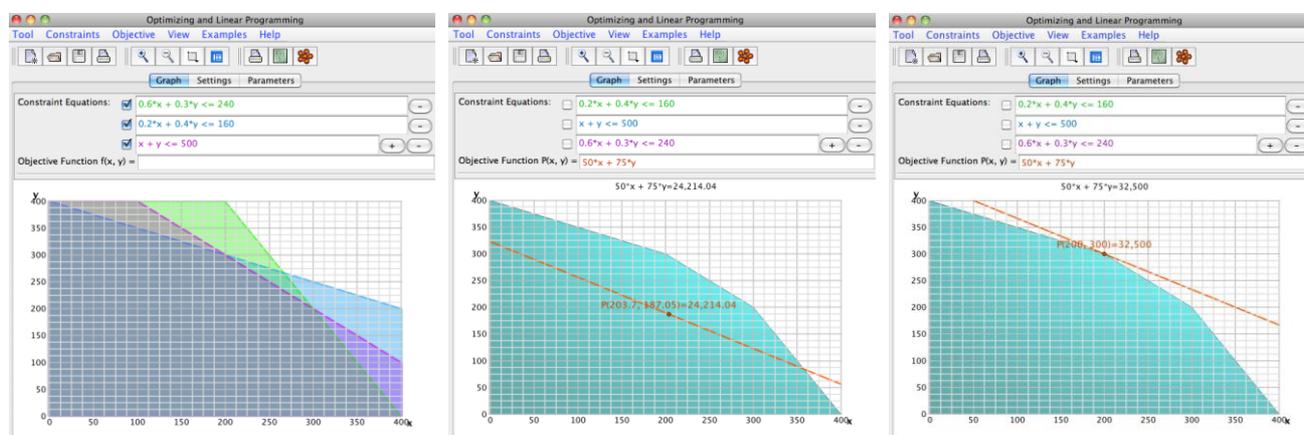


Figure 2. Searching for optimal profit for video game systems production.

Active Learning—The instructional materials are designed to promote active learning and teaching centered around collaborative investigations of problem situations followed by teacher-led whole-class summarizing activities that lead to analysis, abstraction, and further application of underlying mathematical ideas and principles. Students are actively engaged in exploring, conjecturing, verifying, generalizing, applying, proving, evaluating, and communicating mathematical ideas.

Multi-dimensional Assessment—Comprehensive assessment of student understanding and progress through both curriculum-embedded formative assessment opportunities and

summative assessment tasks support instruction and enable monitoring and evaluation of each student’s performance in terms of mathematical practices, content, and dispositions.

CONTENT FOCAL POINTS

TCMS features a coherent and connected development of important ideas drawn from four major branches of the mathematical sciences as reflected in Figure 3.

<i>Transition to College Mathematics and Statistics</i>	
Unit 1	Interpreting Categorical Data develops student understanding of two-way frequency tables, conditional probability and independence, and using data from a randomized experiment to compare two treatments.
Unit 2	Functions Modeling Change extends student understanding of linear, exponential, quadratic, power, circular, and logarithmic functions to model quantitative relationships and data patterns whose graphs are transformations of basic patterns.
Unit 3	Counting Methods extends student ability to count systematically and solve enumeration problems using permutations and combinations.
Unit 4	Mathematics of Financial Decision-Making extends student facility with the use of linear, exponential, and logarithmic functions, expressions, and equations in representing and reasoning about quantitative relationships, especially those involving financial mathematical models.
Unit 5	Binomial Distributions and Statistical Inference develops student understanding of the rules of probability; binomial distributions; expected value; testing a model; simulation; making inferences about the population based on a random sample; margin of error; and comparison of sample surveys, experiments, and observational studies and how randomization relates to each.
Unit 6	Informatics develops student understanding of the mathematical concepts and methods related to information processing, particularly on the Internet, focusing on the key issues of access, security, accuracy, and efficiency.
Unit 7	Spatial Visualization and Representations extends student ability to visualize and represent three-dimensional shapes using contour diagrams, cross sections, and relief maps; to use coordinate methods for representing and analyzing three-dimensional shapes and their properties; and to use graphical and algebraic reasoning to solve systems of linear equations and inequalities in three variables and linear programming problems.
Unit 8	Mathematics of Democratic Decision-Making develops student understanding of the mathematical concepts and methods useful in making decisions in a democratic society, as related to voting and fair division.

Figure 3. TCMS units and their sequence.

ACTIVE LEARNING AND TEACHING

Each lesson includes 2–5 mathematical investigations that engage students in a four-phase cycle of classroom activities, as described below—*Launch, Explore, Share and Summarize, and Check Your Understanding*.

The Launch phase of a lesson promotes a teacher-led class discussion of a problem situation often supported by a video clip and of related questions to think about, setting the context for the student work to follow and providing important information about students’ prior knowledge. For example, “How can you assess the risks of behaviour such as using tanning beds?” or “How is a credit card number sent securely when you busy music online?”

In the second or Explore phase, students collaboratively investigate more focused problems and questions related to the launch situation. This investigative work is followed by a teacher-led class discussion in which students summarize mathematical ideas developed in their groups, providing an opportunity to construct a shared understanding of important concepts, methods, and supporting justifications. Finally, students complete a formative assessment task related to their work.

Each lesson includes homework tasks to engage students in applying, connecting, reflecting on, extending the concepts and methods of the lesson, and reviewing previously learned mathematics and statistics and refining their skills in using that content.

FORMATIVE AND SUMMATIVE ASSESSMENT

Assessing what students know and are able to do is an integral part of *Transition to College Mathematics and Statistics*. There are opportunities for formative assessment in each phase of the instructional cycle. Quizzes, in-class tests, take-home assessment tasks, and extended projects are included in the teacher resource materials for summative assessments.

PRELIMINARY EVALUATION FINDINGS

The following findings are based on multiple beginning and end-of-year measures in six high schools located in Colorado, Kentucky, Michigan, New York, and Texas:

- Three schools in which students had completed a single-subject sequence of algebra, geometry, and advanced algebra from different publishers;
- Three schools in which students had completed a three-year integrated mathematics program.

TCMS End-of-Year Attitude Survey

Among the major findings revealed by the end-of-year attitude survey:

- Students across all six field-test schools generally found the statistics and coding/cryptography units to be most interesting, noted the mathematics was reasonable to understand, and found the contexts very interesting and something they could relate to.
- Students from schools using conventional single-subject courses couched many of their comments in terms of the TCMS real-world problems in contrast to the non-context experiences they generally encountered in their previous courses.

ITED Highest Level Quantitative Thinking Test

The ITED Quantitative Thinking Test for end of grade 12 consists of 40 items focusing on thinking, reasoning, and problem-solving skills.

- All schools showed pre-post gains greater than expected (national) norms.
- One school using the integrated program made significant gains at the $p = .05$ level, as did one school using a conventional curriculum.
- Across all schools, gains were particularly notable for students in the first and second quartiles.

Teacher Interviews

Field-test teachers were interviewed throughout the TCMS field test. At the end of the school year, they were asked: “What do you see as the major strengths of TCMS?” Responses included:

- TCMS reaches out to a select population of students that we previously had nothing to offer them.

- TCMS is the perfect class for collaborative learning. Students learn to actually read in a math class, they learn how to make mistakes and learn from them as opposed to being discouraged by them, and they also develop a deeper understanding of the mathematical material since the topics are all in a real-world context.
- This course really made both the teacher and students *think* about the mathematics being taught and learned. It gave a lot of students who were unsuccessful in Algebra 2 an opportunity to be successful. They enjoyed most topics and the contexts were very engaging. Many of my students left at the end with a view of mathematics as being useful.

Student Interviews

Students were similarly interviewed throughout the field testing. At the end of the school year, they were asked: “Would you recommend this courses to students (juniors) considering a math course to take for next year?”

- Yes, because it seems like we learn things that are much more applicable to daily life and careers than other math classes.
- This course takes difficult skills and ideas that I may have misunderstood in previous courses, reviews and extends them.
- I think this class will prepare you for college with the layout of the class you get to learn many different types of math.

Analysis of student placement and performance in their first college mathematics or statistics course is in process.

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